

A SURVEY OF GROWTH AND YIELD MODELLING METHODOLOGIES

1. Background & Introduction

1.1 Background

This survey of the varieties of growth and yield models and modelling methodologies currently used in forest management serves as a background paper to the Eastern Ontario Model Forest Project entitled "Hardwood Crop Planning - Relative Density Guidelines for Eastern Ontario Hardwood Stands". The survey report will enable Model Forest Partners to identify modelling methodologies that are suitable for use in the Model Forest. The project is relevant to a number of Eastern Ontario Model Forest (EOMF) objectives.

A dominant forest management question on the EOMF is how to escape from the cycle of low stand value that many of the region's hardwood and mixed wood stands are caught in. The problem is that many of these stands are second-growth resulting from past poor management practices. The timber in them is of low quality and low value. The returns from stand improvement work will therefore be low, and so little of this work is done. Hence, the stands remain low-valued.

Many computer models have been developed to simulate alternative management approaches on stands and forests. One goal of the hardwood crop planning project is to identify modelling approaches and models that are suitable for designing effective management strategies for the EOMF. This review of modelling approaches will describe the broad classes of models available and discuss some of their advantages and disadvantages.

1.2 Introduction

There has been a progression in growth and yield estimation methodologies that mirrors the development of other scientific fields. There are many factors driving such developments. Certainly the development of computer technology has enabled people to conceive of and build more complex and demanding estimation approaches. One might also argue that increased cost of timber management and the need to retain as much public trust as possible are necessitating better growth and yield estimation procedures so as to improve forest management decision-making.

Early methods of estimating growth and yield typically used two or three stand level variables as independent variables. For example, empirical yields for a given species were often estimated from site quality, stand age, and stocking. As statistical methodology developed, more independent variables could be added to improve estimates.

Subsequently, researchers modified the whole-stand approach to derive separate growth and yield estimates for stand components. The first efforts in this direction subdivided the stand into size classes. The growth and yield of each size class was estimated and aggregated to the stand level. Compared to whole-stand methods, this approach required additional calculations to estimate the number of stems moving from one class to the next largest one(s).

More recently, growth and yield estimates have been made for each tree in the stand. The premise

for using this level of detail is that more precise estimates can be made.

A corollary of the drive to estimate stand development using more detailed data is that such models can be applied to a wider variety of stands. For example, an empirical yield curve is useful when the subject stand is similar to the stands on which the table is based. But, empirical yield tables give inaccurate results when used on stands with size class distributions that differ from the one reflected in the tables. However, if growth and yield is estimated separately for each tree, then differences in size and species distribution can be taken into account in the estimation of the growth and yield of a stand. Hence, the individual tree modelling methodology can be applied to a wide range of stands. Perhaps the greatest advantages of moving to approaches based on individual trees are realized when one is estimating the development of uneven-aged stands.

Another advantage of using individual tree measurements is that attributes besides timber volume can be assessed. One such stand characteristic is the mix of products in a stand and their quality. Tree level data are helpful in this regard because the presence of one or two very high quality red oak or black cherry trees per hectare can have much more impact on the value of a stand than significant increases in low or medium quality timber volume. There is also scope for linking estimates of wildlife habitat potential, recreational opportunities, and even general aesthetic appeal to these forest management models if we can link these properties to stand characteristics.

The next section of this report will describe various measures of stand density. One or more of these are used in almost all modelling approaches, including the models described in chapter 6. A common stand-level growth modelling technique is to estimate the maximum potential rate of growth and then reduce it according to the degree to which the site is under-utilized, as measured by stand density. Individual tree growth is estimated in a similar fashion, with the actual tree growth rate being derived by reducing the tree's potential growth according to its competitive status. This has led to the development of a variety of relative density measures.

Chapters three, four, and five review broad growth and yield modelling methodologies. These methodologies have been sub-divided into three main classes (as per Davis and Johnson, 1987):

- i) those based on stand-level characteristics;
- ii) those based on diameter classes; and
- iii) those which make use of individual tree data to produce stand level estimates.

One chapter will be devoted to each methodology class, in the order listed above. Throughout this discussion, the advantages and disadvantages of alternative methodologies will be highlighted to the extent possible. The intent is to describe and compare methodologies as opposed to individual models. Since the author has not personally used many of the models reviewed, this section of the report has been supplemented by a variety of written reports.

Section six will survey five widely used models and one important modelling methodology. This will provide some concrete examples of the methodologies described above. Section seven contains some concluding observations.

2. Assessing Stand Density and Growing Space

Trees require light, nutrients, water, and space to grow. The amount of nutrients, water, and light available per hectare are determined by the characteristics of the site. The growth rate of the stand is determined by the degree to which the stand uses the available nutrients, light, and space. However, the growth rate of individual trees is dependent on the amount of nutrients, light, and space available per tree, which is in turn dependent on the number of trees per hectare. Estimating these relationships and translating the estimates into growth estimates is at the heart of growth and yield modelling.

Where stand development is concerned, the important issue is to determine the degree to which a site is occupied by the stand, which is known as stand density. Even when individual tree development is of interest, one can infer from stand density the amount of growing space available to individual trees.

The general concept of stand density is easily understood but it is difficult to define and has proven difficult to measure. Spurr (1950, pg. 275) viewed an ideal density measure as being one:

- i) that is independent of age,
- ii) that is independent of site, and
- iii) that is easy to measure objectively.

Independence with respect to age and site quality is difficult to attain. Clutter et al. (1983, pg. 70) state that "a measure of stand density should be easily and objectively measurable, biologically meaningful, and highly correlated with both stand growth and yield."

2.1 Trees per Hectare, Volume per Hectare, and Average Tree Diameter

Some attempts have been made to use the number of trees per hectare, average tree diameter, and the volume per hectare as density measures. These variables have proven to be unreliable except in very specific circumstances, such as in plantations that have been established at a known spacing.

The main problem is that these variables are often confounded by age and site. The number of trees per hectare in a normal stand decreases with age and increases with site quality. Furthermore, Spurr (1952, pg. 276) notes that "One fully stocked stand may have several times as many stems per acre as another of the same age on the same site." Thus, the number of trees per hectare is not a reliable guide to stand occupancy. The same problems arise with the use of volume per hectare and average tree diameter.

2.2 Stocking

Stocking is a measure related to density but is not by itself a measure of density. Rather, stocking is the degree of site occupancy relative to some reference level. Usually the reference level is the density that provides for "optimum" or "best" growth (Husch et al., 1982). Thus, a fully stocked stand can be expected to exhibit optimum growth, while stands that are under-stocked or over-stocked will

exhibit less than ideal rates of growth.

Stocking lends itself to use in situations where the actual stand growth or yield is calculated as some proportion of the potential stand growth or yield.

A number of authors have developed stocking guides. Figure 2-1 is the guide developed by Gingrich (1967) as reproduced on page 335 of Husch et al. (1982). The chart relates basal area, average tree diameter and trees per hectare and relates these variables to stocking percentage. Note that a rather wide range of stocking levels are considered to be fully stocked. This reflects difficulty in defining exactly what full stocking is.

2.3 Basal Area

One of the most commonly used measures of density is basal area. This is the cross-sectional area of the stand measured at breast height. It is often calculated by assuming each tree is circular, calculating the basal area associated with each diameter, multiplying the per tree basal area by the number of trees of each diameter, and then summing the basal area in all size classes.

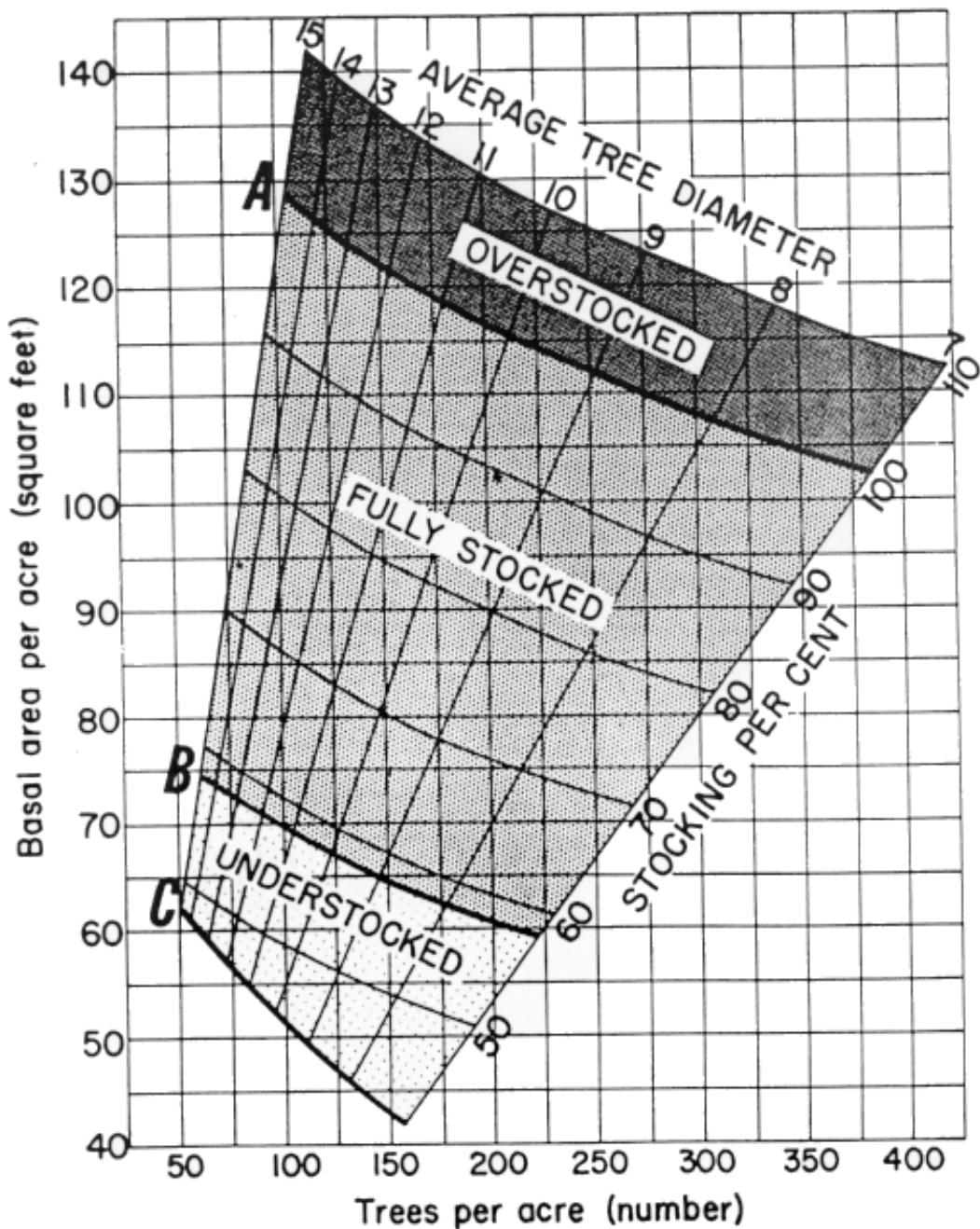
Basal area is not an ideal measure, but it is one of the most commonly used. Spurr (1950) viewed it as the standard against which other measures should be assessed. Basal area is less than ideal because it is somewhat age-dependent - basal area increases with stand age in young stands but levels off thereafter and becomes relatively constant in the mature stand. The basal area that a site is capable of supporting varies with species (see section 6.4: SILVAH) and to some extent with site quality. However, Spurr (1950, pg. 276) concluded that "For a given species growing in a stand of a given age on a given site, ..., basal area accurately measures the degree of utilization of the area...".

Clutter et al. (1983, pg 71) were more reserved in their assessment of basal area, noting that "it has proved useful in yield estimation" in unthinned even-aged stands or plantations of a given age and site and in uneven-aged stands with a reasonable stable age distribution. They also note that growth and yield estimates are often improved when both basal area and the number of trees per hectare are used as independent variables.

2.4 Stand Density Index

Basal area can be calculated from the average diameter and the number of trees per hectare. Some researchers have devised different indices of density using these two component variables of basal area.

Figure 2-1.
Relation of basal area, number of trees, and average tree diameter to stocking percent (Source: Gingrich, 1964)



One of the best-known is the stand density index developed by Reineke in 1933. He found that a straight line relationship often arose when, in even-aged stands, he plotted the number of trees per hectare against the diameter of the average tree on logarithmic cross-section paper. Moreover, for each of the several softwood and hardwood species he examined, Reineke found that the slope of the plotted line approximated -1.605.

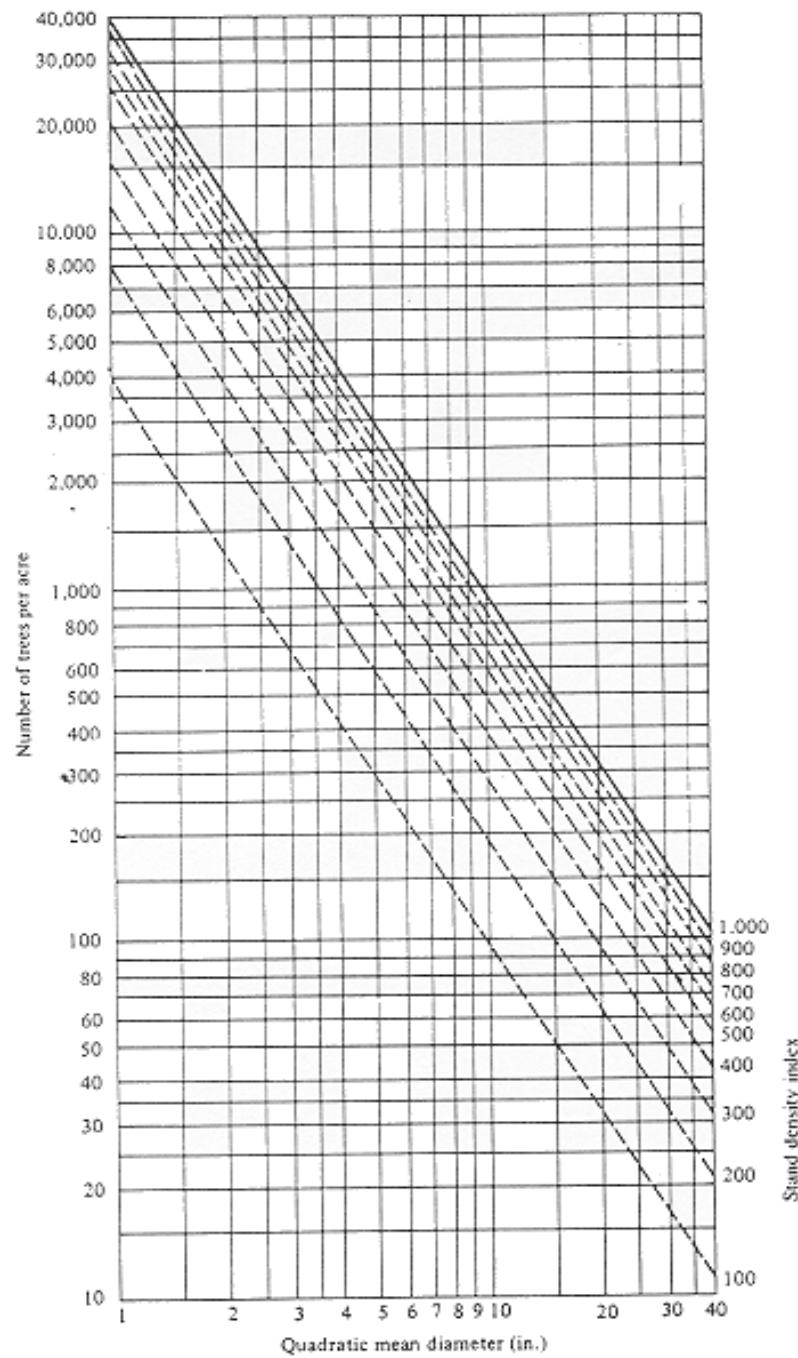
Figure 2-2, reproduced from Davis and Johnson (1987, pg 83), shows a series of stand density index lines. Each line has a unique index value equal to the number of trees per hectare when the quadratic mean diameter is 10 inches. For example, a stand with 690 trees at an average of 3 inches dbh has a SDI value of 100. So does a stand with 100 trees at 10 inches dbh. Accordingly, both stands have the same density. Each species has a maximum SDI value that can be attained. If the maximum SDI value is 500 for the species in the two stands described above, then both stands would be 20% stocked.

Reineke's stand density index has advantages in that the data needed to calculate it are readily available from the inventory. Site and age do not need to be estimated. Also, the SDI value shows less dependence on age than basal area (Spurr, 1950, pg. 280). Spurr (1950, pg. 280) also found evidence that site index was correlated with SDI.

The disadvantages of SDI are that it is a more complicated measure than basal area and it is not suitable for use in uneven-aged stands. Spurr examined 180 fully stocked Douglas-fir stands and found that the slope of trees per hectare against quadratic mean diameter was close to -1.605 in about 1/3 of the cases, but it was flatter in even more cases. In some cases, the slope was steeper. Thus, Spurr concluded that "the stand density index itself is not a hard and fast relationship but merely an expression of an average relationship which should hold for the average of a number of stands, but not necessarily for any given stand" (pg 280).

Spurr (1950, pg 282) concluded that basal area was to be preferred to SDI as a measure of density, whereas (Husch et al., pg. 330) felt that the SDI was "an additional valuable parameter in describing a stand, especially in yield table construction". None of the models described in section 6 use Reineke's SDI.

Figure 2-2.
Example of a stand density index guide.
(Source: Davis and Johnson, 1986)



2.5 Relative Stand Density

Drew and Flewelling (1979) derived a stand density index similar to Reineke's that was intended to be used by managers to help control the density of plantations. This index is calculated as the number of trees per hectare in the subject stand divided by the maximum number of trees per hectare attainable in a stand with the same mean tree volume as the subject stand. In other words, the relative density of a subject stand is the ratio of its number of stems to the maximum number of identically sized trees that could be grown on the site.

These concepts are illustrated in a density management diagram such as that for Douglas-fir shown in Figure 2-3, taken from Drew and Flewelling (1979). The x-axis shows trees per unit area and the y-axis shows mean tree volume. A metric and an English set of axes are shown in Figure 2-3. Three straight downwards sloping lines are drawn at relative density values 0.15, 0.55, and 1.0. The 1.0 line tracks the empirically fitted maximum observed in nature. Total growth per acre levels out and competition induced mortality begins at a density of 0.55 and crown closure is observed at a density of 0.15. Management that maintains stands at densities of 0.55 will promote maximum growth. The graph also contains a set of downward sloping lines pertaining to different tree heights (in feet) and a set of gradually upward sloping lines pertaining to different tree diameters (in inches). These are intended to help managers bring tree size into consideration when planning thinning regimes. This measure appears to be most appropriate in plantation management but has not been used in any of the models described in section 6.

2.6 Tree-Area Ratio

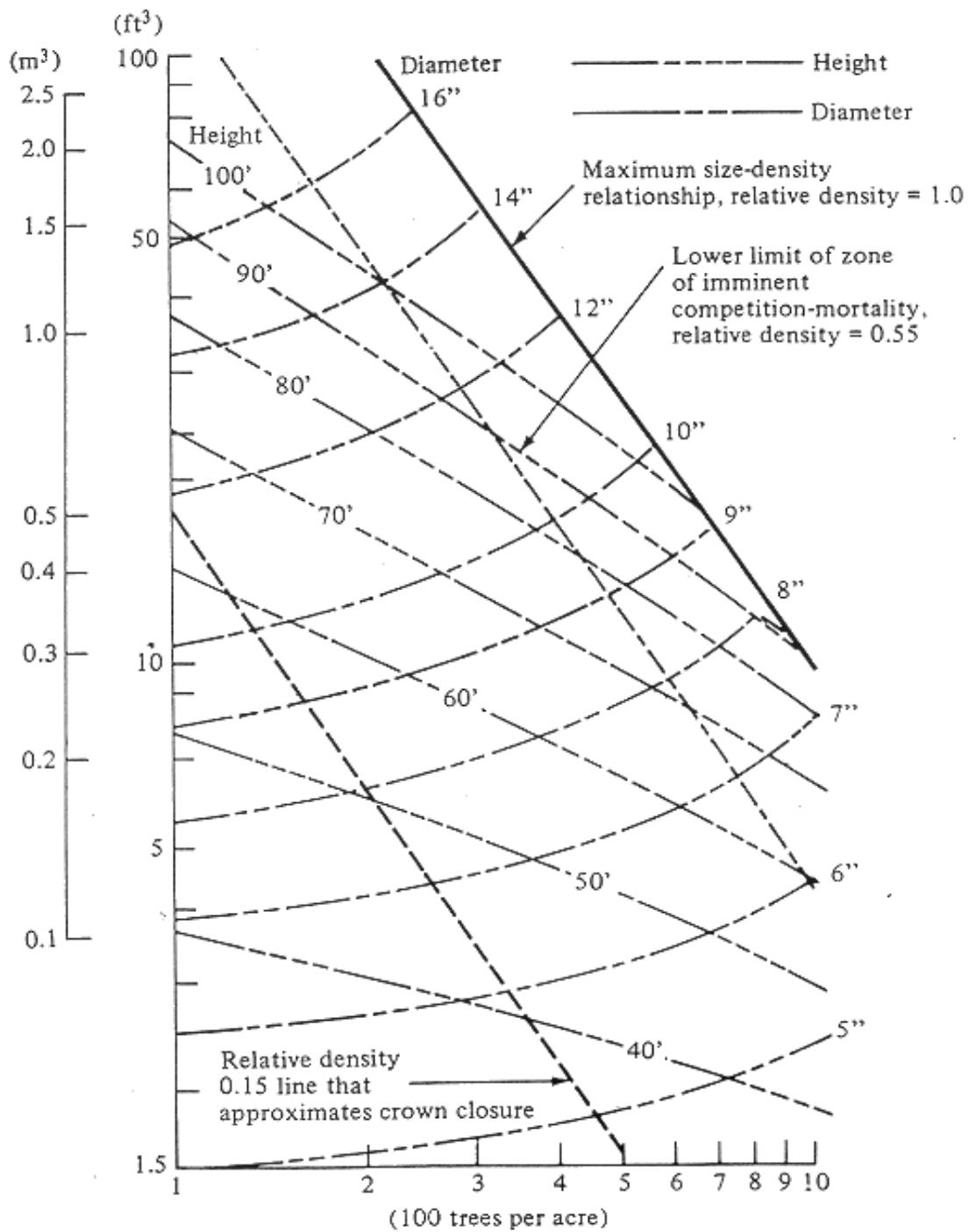
Direct attempts to measure the area occupied by trees began as early as 1914 when researchers hypothesized that the area occupied by a tree was correlated to the horizontal area of its crown. This idea motivated the use of aerial photographs to estimate stand volume. However, it is difficult to accurately define the area occupied by a tree. Physiologists have discovered that root competition occurs in plantations before crown closure occurs. Needless to say, assessing the degree of root competition poses great problems.

Chisman and Schumacher (1940) proposed that the area occupied by a tree is related to its dbh by a second-degree parabola. They postulated that an equation of the following form could be used to express the relationship between the dbh's (D) of trees on a hectare, the number of trees per hectare (N) and the tree-area ratio (Y):

$$(2.1) \quad Y = aN + b\sum D + c\sum D^2$$

where a, b, and c are constants. Y represents the percent space occupied on a per-hectare basis.

Figure 2-3.
 Douglas-fir stand density index as a function
 of number of trees per acre and quadratic mean diameter.
 (Source: Davis and Johnson, 1986)



Spurr (1950, pg. 285) observed that the term $\sum D^2$ is by definition equal to 183.3465 times the basal area per hectare (B). Thus, the equation could be written as:

$$(2.2) \quad Y = aN + b\sum D + c'B$$

where $c' = 183.3465$ times the constant "c" in equation 2.1. From 2.2, it is apparent that the tree-area ratio equals the basal area corrected for trees per hectare and the sum of their diameters. The basal area term contributes a great deal to the value of Y. In fact, Chisman and Schumacher (1940) cited an example in which the basal area factor accounted for 80% of the tree-area value. Spurr (1950, pg. 285) continued by noting that the two correction factors variables, number of trees and sum of diameters can be used to calculate the average tree diameter. Thus, the tree-area ratio is closely related to the stand density index.

Spurr (1950, pg. 284) noted that the chief advantage of the tree-area ratio is that it is largely independent of age and site and can be used in even and uneven-aged stands. It has also proven applicable in stands of timber so open that the stand density index was unusable.

However, there is no proof that the relationship between area occupied and diameter should be a second-order parabola. Secondly, factors such as a tree's competitive status (i.e. dominant, co-dominant, intermediate, or suppressed) can modify the relationship between growing space and diameter. Thirdly, the equations must be determined from plot data and thus the exact equation will depend on the species, stocking, and stand history. This also means that the user must ensure that the equation is suited to the subject stands. Husch et al. (1982, pg 332) suggest that when the plot data used to derive the equation come from fully stocked stands, then the application of the equation to another stand will reflect the proportion of full stocking in that stand. When a range of stands are used to develop the equation, the tree-area ratio equation describes the occupancy in average stands and when data pertaining to a subject stand are substituted into the equation, the stocking of this stand is compared to that of the average stand.

Spurr (1950, pg 285) concluded that while the tree-area ratio was a valid measure of density, basal area was to be preferred, for many of the reasons he cited in connection with the stand density index. Clutter et al. (1983, pg 77) observed that the method has rarely been used in yield estimation because it does not seem to offer significant advantages over other density measures. Husch et al. (1982, pg 332) contended that the measure "is firmly established as a useful measure of stand density".

2.7 Crown Competition Factor

More recently, Krajicek, Brinkman, and Gingrich (1961) proposed a measure of density based on the crown sizes of trees on a site. The crown competition factor (CCF) computes the area of the crowns of trees on a hectare as a percentage of the one hectare area. An important assumption is that a tree of a given diameter has a crown size equal to that of an open-grown tree of the same dbh.

Krajicek et al. (1961) set out the following steps to develop a CCF formula for a species or species group (on a per acre basis):

1. Measure the crown width and dbh of a sample of open-grown, undisturbed trees.
2. Use least squares regression to derive an equation relating crown width (CW) to dbh (D) (i.e.

$$(2.3) \quad CW = a + bD$$

where a and b are constants).

3. Derive a formula that expresses the crown area of individual trees as a percent of 1 acre. The measure produced is the maximum crown area (MCA), which is the maximum proportion of an acre that crowns of a tree of a given dbh can occupy. For example,

$$(2.4) \quad MCA = \frac{\pi(CW)^2}{4} \times (100/43,560)$$

In this equation, the first term is the conversion of crown width (CW) to crown diameter. This is then divided by the number of square feet in an acre and multiplied by 100 to give a percentage figure. Equation 2.4 can be re-written as

$$(2.5) \quad MCA = 0.0018(CW)^2$$

4. Substitute equation (2.3) into (2.5) and expand the squared term to produce:

$$(2.6) \quad MCA = 0.0018(a^2 + 2abD + b^2 D^2)$$

5. The CCF is the sum of the MCAs of all trees on an acre. Calculate it by measuring the diameter of all trees on the hectare and use equation calculate the MCA of each tree. The CCF is the sum of the MCAs.

A CCF of 100 indicates a situation where crown closure has occurred, if all trees are evenly distributed over the acre. CCFs that exceed 100 can be obtained in denser, multi-level stands of tolerant species, in part because the crowns will be smaller than those of open-grown trees.

Spurr (1950, pg 282) reported that crown closure is poorly correlated with basal area and other measures of density. Therefore, any attempts to estimate density based on canopy cover should use additional variables. Although the CCF may appear to be a measure of crown closure, Krajicek et al. (1961) emphasize that it "estimates the area available to the average tree in the stand in relation to the maximum area it could use, if it were open grown".

The CCF appears to be well-accepted as a density measure and it is used in the PROGNOSIS model (section 6.2). One of its advantages is that it can be used in even-aged and uneven-aged stands. It is also largely unrelated to site and age.

2.8 Point Density

Point density refers to the apparent density of a stand **at a particular point in the stand**. These measures are used to measure the degree of competition faced by a particular tree in the stand and such measures are used in individual tree growth and yield models (See section 5) to estimate tree growth, mortality, and even the extent of regeneration (Clutter et al., 1983, pg. 81).

There are a variety of point density measures. Most are based on the notion that each tree is surrounded by a "competition-influence zone" where the presence of other trees or the crowns of other trees creates competition. Point density measures are based on the proportion of this area that is shared with other trees. The variety of measures stems from different ways of assessing the "competition-influence zone" and from different approaches to assessing the level of competition within this zone.

A commonly-cited example of a point density index was suggested by Hegyi (1974). His competition index for subject tree i , (CI_i), is the ratio of the dbh of competing tree j (D_j) divided by the dbh of tree i (D_i), weighted by the distance between the two tree ($DIST_{ij}$) and summed for each of J competing trees. The equation is:

$$(2.7) \quad CI_i = \sum_{j=1}^J \frac{D_j/D_i}{DIST_{ij}}$$

Clutter et al. (1983, pg. 83) report that one researcher found Hegyi's index to be as highly correlated with loblolly pine dbh and height growth as more complex measures based on overlapping areas. In spite of some successes, Clutter et al. (1983, pg 83) conclude that the usefulness of point density measures to predict growth in individual tree growth and yield models is questionable, since models using point density are frequently no better at predicting tree growth than similar models without point density. One possible reason is that microsite differences and root grafting confound the predictions.

2.9 Effects of Density on Growth and Yield

The impact of stand density on growth and yield has been heavily studied for a long time by foresters. Numerous spacing and thinning trials were conducted with this objective in mind. Consequently, there is wide agreement around several general principles.

Spurr (1950, pg. 291) and Clutter et al. (1983, pg 65) conclude that height growth is not affected by stand density over wide ranges of density. Only in very open stands and very dense stands is height growth reduced. For this reason, site quality is widely assessed on the basis of tree height, as with the site index measure.

On the other hand, the diameter growth of trees is inversely related to stand density. A large number of studies support this finding too. Clutter et al (1983, pg 66) note that competition begins to reduce diameter growth at "surprisingly" low densities. Clutter et al (1983, pg 66) and Spurr (1950, pg 293)

both note that diameter growth may be reduced by heavy shrub and herbaceous cover, especially on drier sites.

Finally, Clutter et al (1983, pg 67) and Spurr (1950, pg 294) both conclude that density may have some effect on tree form, but the effect is usually marginal in the ranges of density found in most forests.

Density also affects net volume growth rate per hectare. In uneven-aged stands, the general relationship between growing stock volume (i.e. density) and net annual growth is as shown in Figure 2-4, whereas Figure 2-5 depicts the growth rates of dense and widely-spaced stands against stand age. In both cases, volume growth increases with density up to a point that is likely close to full stocking. In over-stocked stands, volume growth is reduced.

However, many studies have also shown that the growth of under-stocked stands tends to exceed that of fully-stocked stands, so that the volume of under-stocked stands tends to approach normality. Similarly, over-stocked stands tend to have reduced growth and also approach normality. This convergence towards normality by under-stocked stands has been captured in Gerhardt's equation:

$$(2.8) \quad g = dG(1 + k - kd)$$

where g = periodic growth of the subject stand

G = periodic growth of the normal stand of the same age
and site

d = the percent stocking

k = a constant varying by species.

The constant k is generally between 0.6 and 0.7 for intolerant species, 0.8 to 0.9 for mid-tolerants, and 1.0 to 1.1 for tolerant species. These values imply that tolerant species stands approach normality more rapidly than stands of intolerant species.

In contrast, within wide limits, density does not affect gross growth (i.e. net growth + mortality). The reasoning is that as long as the site is fully utilized, the total amount of growth per hectare is not greatly affected (Spurr, 1950, pg 288). Therefore, the role of density control is to allocate the growth amongst individual trees in a stand.

It is of interest that some measures of density appear to be better correlated with growth than others. Spurr (1950, pg 282) cited two studies where basal area gave better correlations with growth than SDI.

Figure 2-4.
General relationship between net volume growth rate
and growing stock in all-aged stands.
(Source: Clutter et al., 1983)

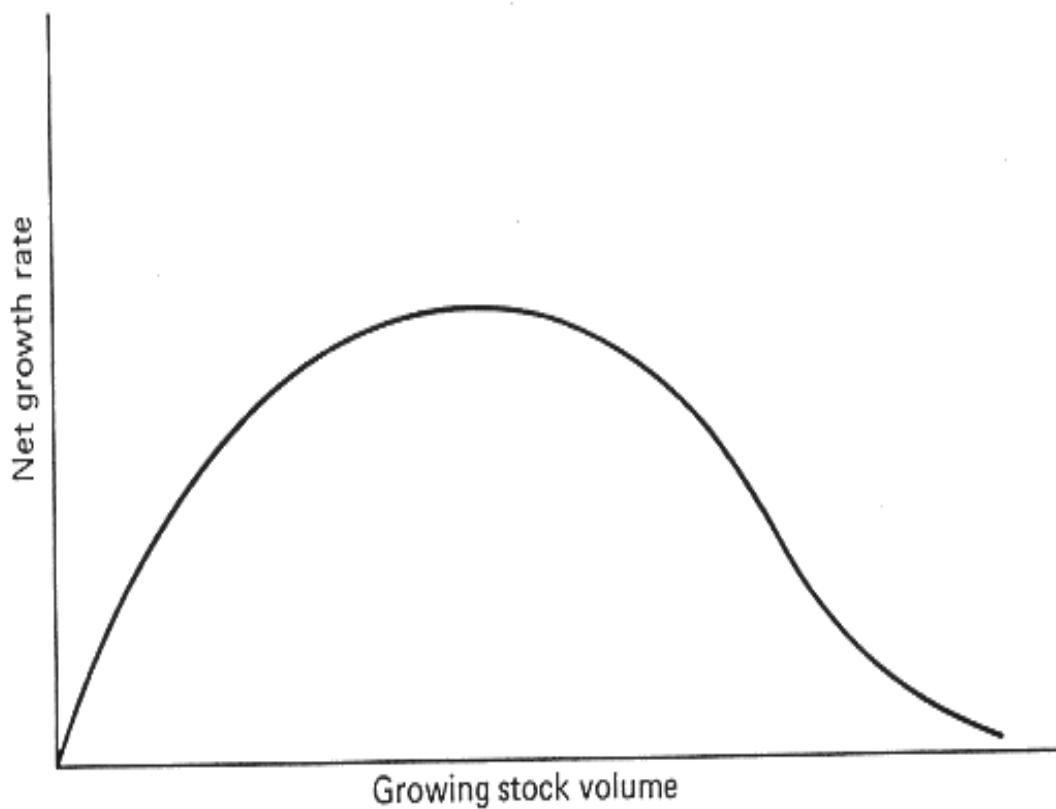
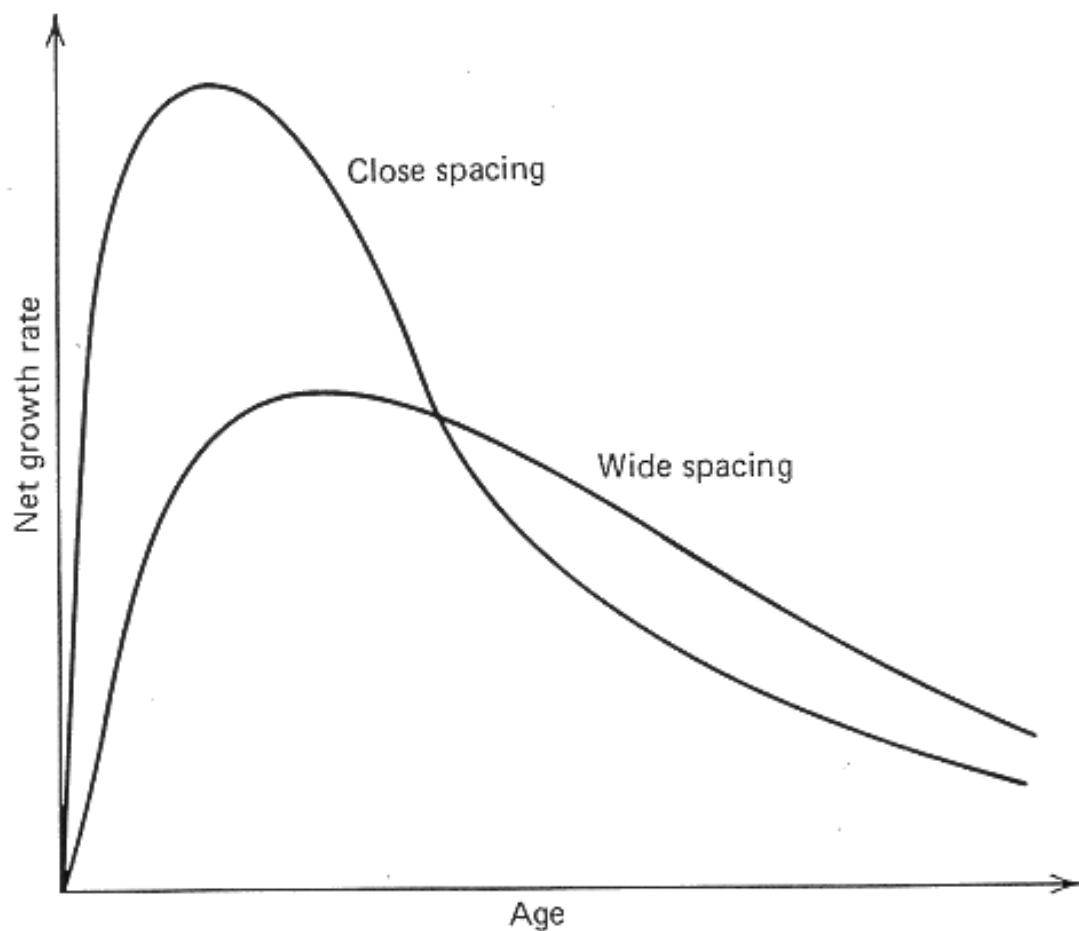


Figure 2-5.
An illustration of stand spacing effects on
net growth rate.
(Source: Clutter et al., 1983)



3. Whole Stand Growth and Yield Estimation Methods

As the title of this section indicates, the estimation approaches described in this section rely on the use of whole stand measures. The most common of these are:

- stand site quality
- stand age
- basal area/hectare
- number of stems/hectare
- average diameter

These methods are best suited for use in even-aged stands that match the characteristics of the stands upon which the original model was built. Where this assumption holds, the methods discussed here are accurate. In fact, the approaches described here are still in very wide use in forests that are predominantly even-aged, especially forests dominated by one species.

A common advantage of these approaches is that the independent variables required to estimate growth and yield can often be measured quickly and inexpensively, in contrast to individual tree models. This issue will be discussed in more detail below.

3.1 Normal Yield Tables

The first stand level yield tables are thought to have been developed independently by Huber and Hundeschagen in 1824 and they have been used in forestry across the world. Yield tables track volume against age for stands of a given species growing on a site of a given quality. Until the late 1930s, most yield tables were made for "normal" stands, which are defined as stands which are fully stocked throughout their lives.

The standard method of constructing normal yield tables is to base them on measurements taken from a sample of fully-stocked stands of different ages on each site class of interest. The analyst sorts the data and removes abnormal plot data, usually by comparing basal area, number of trees, or mean dbh to a pre-determined reference curve of the parameter plotted against age. Then, basal area, total volume, average height, mean dbh, and number of trees are plotted against age for each site class. Graphical or computational methods, or a combination of these, may be used. These data constitute the main part of the table. The analyst may also calculate the board foot yield, the merchantable volume, and other data of importance. Chapman and Meyer (1949) and Bruce and Schumacher (1950) describe normal yield table construction techniques in detail.

There are several main conceptual difficulties associated with normal yield tables. Perhaps the foremost is that the identification of fully-stocked stands is necessarily subjective. Secondly, stands which are fully-stocked at some age were often under-stocked or over-stocked at earlier ages and cannot be expected to remain fully-stocked. Thus a main premise of the tables is often violated. Thirdly, there are very few natural stands which are fully-stocked at any time in their lives. In practice, normal yields must often be adjusted downwards before they can be applied to actual stands.

Finally, many stands have a wider range of tree ages than one finds in normal stands.

Yet despite these limitations, most foresters feel that a place remains in forestry for normal yield tables. They are a useful measure of potential yield and are useful in understanding how stand structure varies over time.

3.2 Empirical and Variable Density Yield Tables

To give foresters a means of more accurately measuring volume in actual forests, empirical yield tables were developed. These are similar to normal yield tables except that they are based on measurements of "average" stands. This simplified construction because it was only necessary to sample typical stands. However, like normal yield tables, empirical tables are based on only two independent variables - age and site - with a single stocking-by- age relationship embedded in the data.

3.3 Variable Density Yield Equations

Many foresters recognized that the applicability of yield tables could be broadened by estimating yield on the basis of more independent variables. It was only in the 1930's that statistical methods became available which permitted them to do this (Spurr, 1952, pg 262). Some of the earliest oft-referenced variable density yield equations were made by MacKinney, Schumacher, and Chaiken (1937) for loblolly pine, Duerr and Gevorkiantz (1938) for northern hardwoods, and Schumacher (1939).

MacKinney, Schumacher, and Chaiken (1937) used age, stand density, site index, and a stand species composition index as independent variables. MacKinney and Chaiken (1939) reworked the same loblolly pine data using a logarithmic approach to estimate yield based on the same independent variables. Duerr and Gevorkiantz (1938) used main stand age, site quality, density expressed in terms of basal area and age, and a merchantability index.

Mulloy (1944, 1947) used age and stand density index to derive a set of equations that could be used to prepare variable density yield tables for mixed wood forests in eastern Canada and for softwoods in central Canada. He also suggested that a separate set of tables be derived for each site type.

Clutter et al. (pg 96, 1983) list five examples of equations of this type. Three of these were made for southern pine species, one for eastern white pine in Maine, Massachusetts, and New Hampshire (Leak et al., 1970), and one for black spruce in Minnesota (Perala, 1971). Each equation derives yield as a function of age, site index, and a measure of density. Basal area was used three times as the density measure, with number of trees and percent stocking being the other two measures. Leak et al.'s (1970) equation for white pine is shown below as an example.

$$(3.1) \quad \ln(V) = 4.32976 + 0.01580S + 2.0785 \ln(P) - 32.7577/A$$

where: V = inside-bark, cubic-foot volume per acre for all trees 3.0 inches dbh and larger to a 3.0 inch top diameter.

S = site index (50 year base) in feet

P = percent of full stocking

A = stand age in years

One can see from the recent publication dates of the cited references that the Schumacher approach was popular until very recently because of its utility and relative ease of use. Clutter et al. (pp 97-99) list other similar equation forms that were used by other researchers as the basis for yield estimation.

Two methods have been most widely used for predicting the future yield of stands (Clutter et al., 1983, pg. 105). These are:

1. Predict the stand density (and values of other variables as needed) at the future age of interest and then plug these values into the current yield equations to solve for future yield. This method assumes that current volume-density-age-site relationships will be maintained into the future. A density development equation is needed to predict future stand growth; if density is tracked in terms of the number of trees per hectare, a mortality equation is required.
2. Use current stand conditions and the length of the projection period as independent variables to solve directly for future yield.

Schumacher was also in the forefront of another key development in growth and yield estimation: the development of integrated growth and yield equations. The problem was that growth equations were often estimated independently from yield equations. The result of this practice was that growth rates were often inconsistent with yields. In part, this was the result of a scarcity of plot remeasurement data that were linked with a useful density measure.

Schumacher and Coile (1960) and Coile and Schumacher (1964) (as well as Buckman (1962) who was working independently) devised and demonstrated a technique that could be used to derive compatible growth and yield equations. The key was to use the principle that the sum of all growth up to a given age equals the yield at that age. Thus, integrating a growth function would produce a compatible yield function (or differentiating a yield equation would produce a compatible growth equation). Clutter (1963) formalized the approach used by Buckman and Schumacher and Coile. In 1972, Sullivan and Clutter refined the technique by showing how separate growth and yield equations could be simultaneously estimated from one data set.

A second type of growth model was pioneered by Richards (1959) and Chapman (1961) using South African slash pine data. The Chapman-Richards growth model is patterned on biological processes. Growth is assumed to be the difference between anabolic growth and catabolic destruction. Anabolic growth is assumed to be proportional to the size of an organism raised by some power and catabolic growth is assumed to be directly proportional to size. The resultant base equation is (3.2):

$$(3.2) \quad dY/dt = aY^b - cY$$

where: Y = size of the organism or population

t = time

a,b,c = constants.

This equation can be solved under the condition that $Y=0$ when $t=0$ to give rise to a sigmoid yield curve. Clutter et al. (1983, pg 125) note that the Chapman-Richards function has been used by many researchers to derive site index curves and was used by Moser and Hall (1969) to model the development of uneven-aged mixed northern hardwood and oak-hickory stands in Wisconsin. The CS-TWIGS equation for potential basal area growth is also based on the Chapman-Richards model (see section 6.5).

4. Diameter Distribution Models

The ability to divide a stand into diameter classes gives a forest manager a potentially more accurate means of estimating growth and yield. In addition, the size class data allow a manager to better estimate the product mix and logging costs of stands.

The forerunner of estimation techniques that use diameter distribution data is known as stand table projection. A stand table is a matrix of numbers of trees by dbh class, sometimes broken into species components if necessary. Ideally, these data should be gathered using a statistically sound sampling process.

The key to using the method is to estimate rates of dbh growth by size class. These can be based on increment cores or on other data. If a ten-year projection is desired, the 10-year dbh growth rates are estimated for each size class and applied to the trees in each class. The new species distribution is adjusted for mortality and an ingrowth component is added to the smallest class.

Exactly how many trees move into the next largest class is determined by the assumptions made concerning current size distribution within each class. It is common to assume that trees are evenly distributed within each class. Then, if 5-cm classes are used, a class with a 10-year growth rate of 2.5 cm will see half of its trees advance into the next class. If the growth rate is 4 cm, 80% of the trees will advance. Adams and Ek (1974) describe a sophisticated model based on this approach (see Section 6.1). The model is intended to enable managers to optimize the economic returns from managed uneven-aged northern hardwood stands in Wisconsin.

If one has the data, one may try to estimate the actual distribution of stems in each class. Furthermore, differential growth rates can be applied within a single class if detailed increment data are available. An alternate approach has been to use a probability function to describe size class distribution. Bailey and Dell (1973) first publicized the applicability of the Weibull function as an expression of size distribution and since then, this has become the preferred approach (Clutter et al., 1983, pg 100).

The shape of a Weibull function is governed by three parameters and the graph of such a function may approach the normal distribution or it may be skewed in either direction.

Clutter et al. (1983, pp 102-105, 110-113) describe the set of equations devised by Smalley and Bailey (1974) for shortleaf pine. Stand height and number of trees per hectare are used to estimate the three Weibull parameters. Additional equations predict individual tree height (based on age, diameter, and number of trees per hectare) and tree volume (a function of dbh and height). Smalley and Bailey (1974) also presented a mortality function, which predicted the number of trees per hectare based on initial planting density and the length of time since planting. Using the mortality equation in tandem with the growth and yield equations allowed the researchers to predict future stand conditions.

5. Individual Tree Models

The most highly refined growth and yield modelling systems operate on the principle that the most accurate estimates require that the development of each tree be modelled. In addition, such models also provide a manager with detailed product output and cost information, and they are also compatible with the monitoring of stand qualities such as biodiversity, wildlife habitat and aesthetic appeal. Note too that the focus of these models is on growth projection, rather than simply the estimation of current yields.

Individual tree models fall into two major categories - distance independent and distance dependent. The latter class of models uses data describing the location of adjacent (i.e. competing) trees, in contrast to the former model group.

5.1 Distance Independent Individual Tree Models

The basic logic used to construct these models is similar to that used in stand table projection. The main premise is that a tree may either die or survive and grow during any time interval. If harvest simulation routines are included in the model, as is often the case, a tree may also be cut.

Tree growth is often estimated by using tree size, species, and site quality to first predict the potential growth of the tree (taken to be the growth of an equivalent open-grown tree) and then reducing the growth by an amount conditioned on the degree of competition faced by the tree. CS-TWIGS (section 6.5) and CACTOS (section 6.6) use this approach. The growth of diameter, height, and the crown are all affected differently by competition and so models often estimate these three components of growth separately.

Davis and Johnson (1987, pg. 137) list three competition indices as examples of the many that have been developed:

1. The ratio of tree basal area to the basal area of the average tree in the stand. As this ratio increases, the tree is assumed to be more dominant.
2. The cumulative basal area of all trees larger than the subject tree. This number decreases with dominance. This is used in the PROGNOSIS model described in section 6.2 and in CS-TWIGS (section 6.5).
3. Krumland (1982) devised an index of competition based on the area of crowns of other trees in the stand measured at a height equal to 2/3 of the subject tree's crown height measured up from the crown base. As this index value falls, the subject tree is assumed to be higher up in the canopy and experience less competition. This measure is used in the CACTOS model described in section 6.6. The data requirements are very high for this index and CACTOS has procedures that allow the user to estimate the values of this variable rather than try to measure the crowns directly.

Other modelling methods do not explicitly calculate actual growth as a percentage of potential growth but are instead based on equations that use the degree of competition faced by the tree as an independent variable.

5.2 Distance Dependent Individual Tree Models

The most complex stand development models use individual tree maps as part of the input data set. Each tree's location is specified in terms of XY co-ordinates. This information is expensive to collect and there do not seem to be any examples of this type of model in widespread use. None of the models reviewed in section 6 are distance-dependent.

Staebler (1951) is credited with deriving the first competition index based on the assumption that each tree is surrounded by a zone of influence within which the presence of other trees and other trees' crowns constitutes competition. Staebler defined his competition index based on the degree which a tree's crown is overlapped by the crowns of other trees. The indices devised by Hegyi (1974) and Bella (1971), which have been "consistent performers", are also described by Davis and Johnson.

Crown dimensions can either be measured or estimated. Measurements costs are high but it is also difficult to accurately estimate the size of a tree's crown, hence both approaches are used.

6.0 Description of Six Specific Models

In this section, six models are described in some detail. The descriptions concentrate on the growth and yield estimation procedures used in each model. Five of these models are widely used. They were chosen because of their popularity and because they represent different approaches to somewhat different management issues. The model formulated by Adams and Ek does not appear to be widely used. However, the approach taken by these authors is very different from that used by other modellers and is worth discussing.

6.1 Adams and Ek's Nonlinear Optimal Programming Model

In 1974, Alan Ek published a paper providing a set of non-linear equations for use in stand projection in uneven-aged stands. Equations for upgrowth into larger diameter classes, mortality, and ingrowth were estimated from northern and central Wisconsin plot data. These stands had at least 50% sugar maple (by basal area) and also contained yellow birch, white birch, northern red oak, trembling aspen, and balsam fir. The quality of the plots ranged from site index 40 to 70 feet at fifty years. These equations were modified by Adams and Ek (1974) and are presented below.

The upgrowth equation estimates the number of stems rising from diameter class D into class D+1 (2 inch dbh classes) during a five year growth period:

$$U_D = 0.0033 X_D^{0.88218} S Y_{1D}^{0.48383} \exp(-0.00286(Y_{21}X_1 + \dots + Y_{2N}X_N))$$

where:

U_D = the number of stems moving to class D+1;

X_D = the number of trees in class D at the beginning of the growth period, for classes D = 1,2,3, ..., N;

Y_{11} , ..., Y_{1N} = the class mid-point diameters at breast height (i.e. 7.0, 9.0, 11.0 inches);

Y_{21} , ..., Y_{2N} = the basal areas of trees with the class mid-point diameters; and

S = site index.

The ingrowth equation estimates the number of stems per hectare entering the smallest measured diameter class (6.0 inches):

$$I = 7.07933 ((Y_{21}X_1 + \dots + Y_{2N}X_N)/(X_1 + \dots + X_N))^{-1.40072}$$

and the mortality equation is

$$M_D = 0.04109 X_D$$

where M_D = the number of stems dying in diameter class D.

The mortality equation simply posits that a fixed percentage of existing stems die each period, regardless of diameter size. Note that species are not explicitly identified in these equations. The

ingrowth equation is an asymptotically decreasing function of the weighted average basal area per tree, implying that there is less ingrowth in denser stands. Upgrowth is an increasing function of site class, the number of trees in the class, and dbh class size. Upgrowth is negatively affected by increased stand density, as measured by basal area.

The r^2 values and standard error estimates of the three equations are shown in the table below. These indicate that the upgrowth and ingrowth equations are more accurate estimators than is the mortality equation.

Table 1.
Regression Statistics for Ek's Stand Projection Equations

Equation	r^2 value	Std. Error
Upgrowth	0.76	8.74
Ingrowth	0.72	23.83
Mortality	0.34	4.06

Adams and Ek (1974) used these equations to construct a mathematical programming model that would solve for the initial stocking level that maximizes the value growth of the stand during each growth period (or cutting cycle). The owner of the stand would be expected to harvest this growth at the end of the cutting cycle, returning the stand to the same optimal initial condition. Thus, maximizing the objective function is equivalent to maximizing the value of the harvest at the end of each cutting cycle. The programming model selects an optimal steady-state forest structure; it does not consider how to convert an existing stand to the optimal structure.

The three sets of constraints in the model are quite simple to interpret. There is a set of constraints to ensure that each class has a non-negative number of stems in it, another set of constraints ensures that the stand basal area must equal some fixed value at the beginning of each cutting cycle, and the third constraint prevents the number of stems in each class from decreasing during the cutting cycle. In fact, the number of stems per diameter class is expected to increase during the cutting cycle and this increase will be harvested.

Adams and Ek (1974) noted that few stands are at an optimal condition. For these stands, the problem is finding the set of actions that will bring these stands to an optimal condition. In general, the objective is to maximize the value of management actions (including harvesting) during the conversion period and thereafter when the stand has reached a steady-state condition. The length of the conversion period, the optimal steady state condition, and the optimal cutting cycle are all inter-related variables which should be determined by the model. At the time of writing, computer hardware and software limitations prevented Adams and Ek from examining the general version of the problem. However, when they set the conversion period equal to two cutting cycles and set the length of the cutting cycle at five years, they were able to solve for the optimal conversion strategy.

6.2 PROGNOSIS

PROGNOSIS (Stage, 1973) was developed to help foresters in the northern Rocky Mountain states estimate stand response to treatment. The model is a distance-independent individual tree model that is intended for use in a variety of stand conditions ranging from old growth to heavy cutover with either single or multiple species.

As input data, the model requires some stand variables and the dbh, height, and crown ratio of each tree. There are four main equations used to project stand development. Of these, the diameter increment equation is the key, since dbh becomes an independent variable in the height growth and crown ratio development equations. The diameter increment equation also includes a random element that can be linked to the degree of error in the inventory data. Finally, the model does not actually calculate dbh increment *per se*, but rather the variable dds, which is the number of square inches of new wood grown at breast height during the projection period. This is readily convertible to basal area or diameter increment.

Davis and Johnson (1987) show a generalized equation for dds:

$$\text{dds} = \text{HAB} + \text{LOC} + b_1(\text{SL} \times \cos \text{ASP}) + b_2(\text{SL} \times \sin \text{ASP}) + \\ b_3\text{SL} + b_4\text{SL}^2 + b_5\text{EL} + b_6\text{EL}^2 + b_7\text{dbh} + b_8\text{dbh}^2 + \\ b_9\text{CR} + b_{10}\text{CR}^2 + b_{12}\text{CCF}/100 + b_{13}\text{BAL}/100 + e_i$$

where:

HAB = habitat type

LOC = geographic location

SL = slope

ASP = aspect

EL = elevation

CR = crown ratio

dbh = diameter at breast height

CCF = crown competition factor

BAL = basal area per acre of trees larger than subject tree

e_i = random element.

The variables HAB, LOC, SL, ASP, and EL are all site characteristic variables that are equal for each tree in the stand. The use of BAL and CCF separates the level of competition in a stand from the degree of competition experienced by an individual tree.

The other types of equations in the model forecast mortality, height growth, and the development of the crown ratio over time. Mortality is modelled as a function of tree size and two density measures.

Davis and Johnson (1987) report that PROGNOSIS is becoming increasingly widely used and new versions are being developed to cover different conditions. They note that the advantages of this model are the breadth of the stand conditions and management treatments covered, and wide availability to industry, U.S. Forest Service staff, and members of the public. The main drawback in 1986 was that PROGNOSIS could only be run on a mainframe computer, but I expect that there are PC versions now. The model is also complex and the user requires some time to become comfortable

with it.

6.3 G-HAT: Growth of Hardwoods after Thinning

G-HAT is a distance-independent individual tree model that was derived from two measurements from 66 permanent sample plots in the Blue Ridge areas of Virginia, Tennessee, North Carolina, and Georgia. The sample stand were unharvested, *even-aged*, mixed hardwoods characterised by mixtures of northern red, white, and chestnut oaks and yellow-poplar. Other important species included black cherry, magnolia, red maple, birch, and black and scarlet oak. The authors (Harrison et al., 1986) caution that the model is only applicable immediately after thinning and should not be applied to stands which have been thinned from above, thinned more than once, or high-graded.

There are four sets of equations: basal area increment, survival, height prediction, and volume prediction. Harrison et al. (1986) and Burk et al. (1986) give more detail regarding the construction of these equations. Since this model is only to be used during the response period after thinning, ingrowth is not considered in the model. A basal area increment equation was developed for each species, each having the general form:

$$G = a + bX_1 + cX_2 + dX_3$$

where

G = periodic annual basal area increment over five years

X_1 = original tree basal area

X_2 = stand basal area after thinning

X_3 = stand basal area before thinning

a, b, c, and d are species specific coefficients.

For all species, coefficients a, b, and d were positive, indicating a positive correlation between basal area growth and initial tree basal area, and initial stand basal area. Tree basal area growth declines as more basal area is left after thinning (i.e. remaining trees grow better at lower stand densities).

Height and volume equations were also developed for each species. Height is modelled as a function of dbh, tree height, and average stand height whereas volume is a function of dbh, height, and various intermediate diameters and heights.

A single survival equation was developed for all species:

$$S = 0.90477 + 0.09523[1 - \exp(-0.7247G)]$$

where S = probability of survival after five years and G = predicted basal area increment. The equation is constructed so that the minimum survival probability is 90.477%, increasing with predicted growth (and therefore tree vigour).

This model differs from others reviewed here in that it is designed for very specific circumstances. The model itself allows the user to simulate the impacts of various approaches to thinning from

below. One approach allows the user to set a desired residual basal area and then invoke an automatic thinning sequence. This automatic sequence first removes undesirable species tree by tree, starting with the smallest size class. The residual basal area is checked after each removal and thinning stops if the desired level has been reached. If all undesirable species have been removed and the residual basal area not attained, the routine re-visits the smallest diameter class and removes trees in reverse order of species desirability. If all trees are removed from the lowest class and the residual target has not been attained, successively larger classes are handled in the same manner until the residual basal area equals the user's objective. Then, the user can project the stand's development for up to ten years.

If the user feels that the auto-thin procedure is inappropriate, the user can remove specific numbers of trees by species and diameter class. The authors warn that the thinning should not deviate far from a thinning from below.

6.4 SILVAH: Silviculture of Allegheny Hardwoods

SILVAH (Marquis and Ernst, 1992) is a very different model than those reviewed above. It allows a user to enter data describing a hardwood stand and the user's management objectives and then SILVAH will generate a recommended prescription. In essence, SILVAH is an expert system, since its prescription setting routines rely on the knowledge and perspectives of experienced forest managers from the Allegheny region. SILVAH also contains a growth and yield simulator, which allows a user to devise alternate prescriptions and simulate the outcomes.

SILVAH was originally developed for use in Allegheny hardwood stands dominated by black cherry. Through additions and alterations, it can now be used in beech-birch-maple, cherry-maple, oak-hickory and transitional stand types. However, Marquis and Ernst (1992) suggest that SILVAH remains best suited for stands on the Allegheny Plateau and should be modified if used elsewhere.

SILVAH allows the user to enter raw cruise data. Timber data are formulated as a stand table. The cruise data may also include information about regeneration and competing vegetation, the intensity of deer browsing, access, operability, and environmental stress. The user is asked to select one of four management objectives, which cover various combinations of timber value maximization and promoting aesthetic qualities (chiefly by limiting harvesting intensity). The two potential objectives which weight timber value maximization rather highly generally lead to even-age management prescriptions, whereas the two objectives in which harvest impacts are minimized generally lead to uneven-aged management prescriptions.

Once the data have been entered, the user may request that the program generate a prescription. The user may then simulate the development of the stand if this prescription is followed. The user may set other prescriptions and use SILVAH to forecast the outcome.

SILVAH contains two sets of growth and yield and mortality equations. The original set of equations pertains to Allegheny hardwoods (Marquis, 1982). More recently, OAKSIM equations (Hilt, 1985) have been added to enable SILVAH to be used in oak-hickory stands. Both sets of equations are similar in construction.

Growth and yield is a function of the subject tree's dbh in relation to the average stand dbh, the tree's relative stand density, species, age, and site index (expressed for oaks only). SILVAH is unique in its use of relative density, which is a ratio of the density of the subject stand in comparison with a fully stocked stand of the same species composition (Marquis, Ernst, and Stout, 1992). Relative density is used because the various species found in the Allegheny forest have significant variations in the number of trees per hectare (and basal area) that constitute a fully-stocked stand. For example, a fully stocked black cherry stand may have 40% more basal area than a fully stocked hard maple stand. In these conditions, relative density is preferred to absolute density measures such as basal area. Tree-area ratio equations have been devised to permit easy calculation of a stand's relative density.

Schuler et al. (1993) compared the predictive capability of FIBER, NE-TWIGS, SILVAH, and OAKSIM in a range of forest types found in the northeast United States. The models' predictive capabilities varied among combinations of forest type and state, with SILVAH proving to be superior in a number of northern hardwoods and Allegheny hardwoods conditions.

6.5 CS-TWIGS: Central States TWIGS

TWIGS is a set of growth and yield projection programs which allows a forest manager to test and simulate the impacts of alternate management prescriptions. There are a number of sets of regional growth and yield equations. Shifley (1987) published the growth and yield equations applicable to the central U.S. states of Indiana, Missouri, and Ohio. These are typical of the TWIGS growth and yield equations and are described below.

The CS-TWIGS growth and yield equations are derived as estimates of potential unconstrained growth modified by competition. Potential tree growth is modelled as a function of site index, tree size, and crown ratio. Constraints were added to prevent trees from growing to unrealistically large sizes. Tree basal area growth was estimated because tree volume estimates are often functions of basal area rather than diameter and because the relationship of tree growth to size is "graphically much more distinct" when based on basal area rather than diameter (Shifley, 1987).

Potential annual basal area growth was modeled as an adapted form of the Chapman-Richards growth function described above:

$$POT = [b_1 TBA^{b_2} - b_1/A^{(1-b_2)} TBA][b_4 + b_5 SI + b_6 CR]$$

where CR = crown ratio class, SI = site index, A = maximum tree size, and TBA = current tree basal area. The first bracketed term of the equation is a function of growth that approaches zero as tree size approaches the maximum. The second bracketed term adjusts growth rates upwards by site quality and crown ratio.

The modifier equation was derived as:

$$MOD = c_0 \{ 1 - \exp[-(c_1/BAL + c_2 D^2)(1 - BA/BAMAX)^{1/2}] \}$$

where MOD is the proportion of potential growth actually achieved. Independent variables include

dbh (D), stand basal area (BA), maximum stand basal area (BAMAX), and the basal area of all trees with a dbh equal or larger than the subject tree (BAL). All coefficients in the MOD equation are positive. A greater proportion of potential growth is obtained if the stand basal area decreases or the tree's competitive position as represented by BAL improves or as tree diameter increases.

Actual tree growth is calculated as POT*MOD.

6.6 CACTOS - The California Conifer Timber Output Simulator

The CACTOS model is an individual tree, distance independent model designed to allow a manager to simulate stand development in the absence of intervention or in response to harvesting or thinning (Wensel et al., 1986). CACTOS is one of the few models that allows the user to estimate ingrowth. The model can be used in young or old stands with multiple species and multiple ages. The predominant species types are generally Douglas-fir, white fir, sugar pine, ponderosa pine, red fir, and incense cedar.

The base tree information consist of species, dbh, total height, height to base of crown, and the number of trees per hectare of the type described in the record. Stand level data include species specific site index values. The user can also enter an optional ingrowth file with data records as per the existing stand. CACTOS also permits the user to recalibrate data if necessary.

CACTOS models height and diameter growth by estimating the potential growth of each dimension and then estimating a competition factor which is used to predict actual growth. Wensel and Koehler (1985) note that the approach used in PROGNOSIS was unsuited to California conditions. In PROGNOSIS, height growth is predicted as a function of diameter and expected diameter growth. However, diameter and diameter growth are more sensitive to density than is height growth in California - thus they are poor predictors of height growth.

Potential height growth is derived from the unconstrained height curve which has the form:

$$H = 4.5 + b_0 S^{b_1} [1 - \exp(-b_2 A)]^{b_3}$$

where A = age, S = site index, and b_0 , b_1 , b_2 , and b_3 are parameters. This equation was converted into an equation that predicted unconstrained five year height growth (G_H):

$$G_H = 4.5 + [c_0 S^{c_1} + c_2 (H-4.5)^{c_3}]^{1/c_3} - H$$

Notice that age drops out of the height growth equation.

The height growth competition factor (C_H) is:

$$C_H = \{d_1/[1 + \exp(4 - d_2 LCR)]\} \times \exp(-d_3 CC_{66}^{d4} PBA^{d5})$$

where LCR is the live crown ratio, CC_{66} is the crown area in the stand at 66% of the subject tree's height, and PBA is the proportion of basal area of that species in the stand. PBA is only used for

ponderosa pine. LCR is assumed to reflect the ability of the tree to respond to release and CC₆₆ is assumed to reflect the degree of crown competition. Since CC₆₆ is difficult to measure, the model computes the variable using data collected to estimate the growth and yield equations and the height to crown base. During the course of simulation, the model will estimate changes in the height to the base of the crown as a function of tree height, dbh, and the basal area of trees larger than 5.5 inches dbh.

Potential diameter growth and the diameter competition factor are calculated in a manner analogous to height growth and height competition. Potential five year diameter growth (G_D) is given as:

$$G_D = [c_0 S^{c_1} + c_2 D^{2c_3}]^{1/c_3} - D^2$$

where D is the current dbh. The coefficients c₀, c₁, c₂, and c₃ have different values than they do in the height growth equation. The diameter growth competition factor (C_D) is:

$$C_D = \exp(d_1 CC_{66}^{d_2} CV^{d_3})$$

where CV is the crown volume. CV is estimated by

$$CV = aD^b H^c LCR^d$$

The mortality function used in CACTOS was taken from a redwood model CRYPTOS (Krumland, Dye, and Wensel, 1977). The authors acknowledge that it may not be entirely appropriate for use with other species and further work was planned in this area. The annual probability of a tree's death is modelled as:

$$p = 1/\{1 + \exp[3.71 - 0.0109RBA + 0.11DBH + 0.000445TPA]\}$$

where RBA is the relative tree basal area (average tree basal area/subject tree basal area) and TPA is the number of trees per acre.

7.0 Conclusions

This survey has covered a wide variety of growth and yield modelling approaches and examined six models and methodologies in current use. This survey clearly shows that there are a wide variety of circumstances in which models are used and issues which they are designed to address. Consequently, there is a diversity of modelling approaches. This abundance of models can prove to be confusing and has led to situations in which model(s) were used in inappropriate forest management circumstances. Many authors have cautioned against using a particular model in conditions foreign to those for which it was intended.

Since no two models were designed to address the same issues in the same region, it is difficult to compare models and modelling approaches. This is reflected by a scarcity of comparisons in the literature (although Schuler et al., 1993 is an exception).

Accordingly, Table 2 summarizes the key features of the major classes of model discussed in this report but does not try to rank them by quality. The upper section of the table indicates what features of the forest can be distinguished by models typical of each model class. Five features of particular relevance to the Relative Density project are shown. Almost all models have age and site quality as endogenous variables, hence these variables are absent from the list of sensitivity factors.

The bottom row of Table 2 briefly outlines the circumstances in which each type of model is applicable. Normal and empirical density yield curves are only accurate for stands of normal and average densities, respectively, and so were viewed as being insensitive to density. None of the whole models were viewed as being sensitive to species in the sense that one model is applicable to a variety of species compositions - instead, one has to use species specific tables (i.e. species specific models). The whole stand models are also insensitive to tree size distribution - again, a particular distribution of tree sizes is assumed at each data point. In addition, whole stand models are incapable of handling uneven-aged stands, which is a significant limitation. Only the individual tree models can adequately model a wide combination of age class structures and species compositions within a stand. Within this class, the distance independent models assume a uniform distribution of trees throughout the stand - a distance independent model is required to differentiate a full range of spatial arrangements within a stand.

Whole stand models are best used for even-aged single species stands that have not received any spacing or thinning treatments. They are generally incapable of forecasting changes in species composition, product yields, and harvest costs and they are only accurate for a narrow subset of general stand conditions. However, they are useful for the conditions for which they were designed and are easy to use, often presented as tables or single equations. Diameter distribution models are generally developed for single species plantations or single cover types, and hence are insensitive to variation in species composition. Models in this class do react to size class distribution and, of course stocking, but they assume stands have a uniform tree distribution. The most flexible models are the individual tree models. However, there is often a significant front-end cost to using these models and the data requirements can be onerous. Authors also caution against using

Table 2.
Summary of Model Characteristics.

		Whole Stand Models			Diameter Distribution Models	Individual Tree	
		Normal Yield Tables	Empirical Yield Tables	Variable Density Equations		Distance Independent	Distance Dependent
Model sensitive to	Stocking	No	No	Yes	Yes	Yes	Yes
	Species composition	No	No	No	No	Yes	Yes
	Tree size distribution	No	No	No	Yes	Yes	Yes
	Species composition x size	No	No	No	No	Yes	Yes
	Tree spatial arrangement	No	No	No	No	No	Yes
Where Applicable		Fully stocked, even-aged stands of same species mix as base stands	Even-aged stands of average stocking and same species mix as base stands, large forest areas	Even-aged stands with same species and size class as base stands	Stands with same species as base stands and definable size class distribution	Stands with uniform tree distribution	Potentially all stands

single tree models to predict growth for period of more than 15 -20 years, which is in contrast to the whole stand and diameter-distribution models. Sometimes this warning is given because ingrowth is ignored in the individual tree models. (Ingrowth is reported to be difficult to predict accurately.) This warning also reflects the nature of forecasting error. These errors tend to be systematic and individual tree models are designed so that they magnify systematic errors.

There is controversy over whether individual tree models provide better estimates of growth and yield. Clutter et al (1983, pg 115) go so far as to note that none of the competition indices used in any of the individual tree models has proven more accurate than basal area as a measure of density. Yet, as I think Clutter et al. (1983) would agree, the chief advantages of using individual tree models have to do with the expanded range of conditions in which these models are valid and the ability of these models to correlate stand characteristics to timber products, wildlife habitat, and other non-timber forest qualities.

There is widespread agreement (Davis and Johnson, 1987, pg 140; Clutter et al., 1983, pg 115; B.C. Ministry of Forests, 1991) that the additional cost of collecting the data to support distance-dependent models is not warranted at the present time.

When selecting a model for use, one must first identify the purpose of using the model and the required level of detail. Models which are unsuited for this purpose should not be considered further. While there is a tendency for model users to want the most sophisticated models, the cost of doing so may be quite substantial. One may have to collect or at least input data which do not help the user to address the issue of interest. Of the remaining eligible models, one may then rank them according to two criteria:

- 1) Is the model suited to the forest type(s) of interest? How much effort will be needed to calibrate the model?
- 2) What is the costs and feasibility of obtaining any data required by the model that are currently unavailable?

Based on these considerations, it is clear that distance independent models are most suitable for the range of conditions encountered on the Eastern Ontario Model Forest. The wide range of species compositions and the importance of uneven-aged management systems preclude complete reliance on whole stand or diameter distribution models. On the other hand, there is no justification for the expense and difficulty associated with distance dependent models at any point during the current five-year Model Forest funding period. However, whole stand models or diameter distribution models can be appropriate for use in plantations and other even-aged stands (particularly conifer) in the EOMF.